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Asset Mix Optimization

Using Python and Google Colab

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# Introduction

Asset allocation is a crucial step in portfolio management. An optimized portfolio mix can enhance return and manage risk. In this documentation, we will introduce an asset mix optimization model optimizing the tradeoff between risk and return. We then run this model on a real-world dataset, and back test the resulting optimized mix to evaluate the strategy’s performance and risk. We will walk through the model and codes in this documentation and end the discussion by comparing the advantages and disadvantages of the approach.

# Access to the Model

The model has been coded in Python computer language and is entirely open source. Model is run on Google Colab integrated development environment (IDE) hosted on virtual machines on the cloud.

Code and Google Colab Cloud IDE:  [https://colab.research.google.com/drive/1Ib5o5Xvz-CAvtpDi6Vufo09eC51\_sRhW#scrollTo=hx72Uu02mnGy](https://colab.research.google.com/drive/1Ib5o5Xvz-CAvtpDi6Vufo09eC51_sRhW%23scrollTo=hx72Uu02mnGy)

Data: <https://drive.google.com/drive/folders/1ZXm2usN4kOki4J3MQGlsKM6sIQPgYgdx?usp=sharing>

Repository: <https://github.com/YaokunLin/ActuaryProgrammer/blob/main/Portfolio_Optimization.ipynb>

System requirement: A google Chrome Browser (91.0.4472.77) installed on the operating system.

# Data Sources

The model is run on the real-world data sets of two Exchanged Traded Funds (ETFs):

* Vanguard Total Stock Market Index Fund ETF Shares (VTI)

<https://ca.finance.yahoo.com/quote/VTI/history?p=VTI>

* iShares Core U.S. Aggregate Bond ETF (AGG)

<https://ca.finance.yahoo.com/quote/AGG/history?p=AGG>

VTI is a passively managed equity index fund, and it seeks to track the performance of the CRSP US Total Market Index.

AGG is a passively managed fixed-income index fund, and it seeks to track the investment results of an index composed of the total U.S. investment-grade bond market.

Historical data set are obtained from Yahoo Finance. We utilized all historically available data including both price and dividend histories.

# Model Principle

The approach follows the following steps:

1. Given a historical timeframe (the look-back window), measure the historical averages, standard deviations and correlations of the asst returns.
2. Calculate the expected returns and return standard deviations of all potential mixes.
3. Calculate the Sharpe ratio of each mix:

1. Choose the mix that generates the largest Sharpe ratio
2. Back test the optimized mix by conducting historical simulations based on a user-defined investment horizon and historical data availability.
3. Evaluate the risk and return performance of the optimized mix against policy requirements.

# Codes and Model Walk Through

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| Import necessary Python libraries and mount the datasets from Google drive then transform the dataset into Panadas data frames for analysis. |

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| User Inputs:   * Started\_amount is the amount of seed money to initiate the portfolio. * Lookback\_horizon is the number of trading days the model looks back when measuring the historical performances of the assets, i.e., the look-back window. * Invest\_horizon is the number of trading days for investment purposes, i.e., investment horizon. * Rebalance\_trigger is the threshold for rebalancing. When any of the asset mix percentages deviates from the optimal mix by this threshold, rebalancing will be triggered. This is used for the historical back testing and simulations.      * Risk\_free\_rate is the risk-free rate. It is used to calculate the Sharpe ratio. And dividends are accrued and compounded at this rate for historical simulations. |

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| Comparison on the return distributions of the assets. The distribution depends the user-defined historical look-back window.  As we can see, the equity return distribution has fatter tails. |

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| Data re-organization. |

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| Visually inspect the relationship between fixed income and equity asset returns by years.  No meaningful relationship is observed. |

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| Create an object class called “asset\_class”; it carries the attributes associated with an asset class, including expected return, return standard deviation and Value at Risks VaRs.  The VaR feature of this object class is not used in the analysis, but its infrastructure has been implemented to the object class. |

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| Create a bond asset class object instance and initiate the attributes by the statistics obtained from the user-defined look-back window. |

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| Create an equity asset class object instance. |

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| Create a cash asset class object instance. This is not used in the analysis; it is for completeness. |

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| Create the “assets correlation” object class.  Created an instance of this class named “bond\_stock\_corr”, it carries the correlation information based on the look-back window.  It seems unnecessary to create this object class when there is only one object instance created, but when there are multiple asset classes to be analyzed, the information will be organized. |

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| Create the “portfolio” object class. The bond and stock objects are at its class-level attributes.  Each possible mix is defined as a “portfolio” object instance.  Asset weights, expected return, return standard deviation and Sharpe ratio are the attributes to this object class.  “port1” is a sample instance. |

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| Optimize the portfolio mix. The mix is chosen such that it maximizes the Sharpe ratio.  A portfolio with the highest Sharpe ratio is considered as the one reaches the optimal level of risk-return tradeoff. |

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| Sharpe ratio is optimized when bond weight = 74.5%. |

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| The upward sloped portion of the hyperbola is the efficient frontier in modern portfolio theory. |

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| Record the optimized portfolio for later use. The portfolio is called “port\_sharpe\_optimized”. |

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| Setup for the historical simulations and back testing on the optimized portfolio. |

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| Create an object class named “portfolio\_value” and define all needed attributes. This is to track the portfolio value at each time step of the simulation.  Two main functions associated with this object class:   * “update\_unit\_price” is to update the asset prices. * “rebalance” is to rebalance the portfolio when the weights deviates from the optimal by a user-defined threshold. Thus, this function is to update the weights. |

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| Create a object class named “asset\_price”. This is to keep track of the asset prices at each time-step. |

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| Create a function named “initialized\_portfolio ”. This is to bring back the portfolio value at the initial state at the beginning of each simulation. |

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| Initialize the portfolio. |

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| Historical simulation and back testing.  The first for loop goes through all the simulations. With each of the simulations, the second for loop goes through each time step of the investment horizon.  The two if statements are to simulate the dividend cash payouts based on the historical dividend yields.  The algorithm then updates the asset prices based on the historical scenarios and updates the resulting portfolio value.  When the asset value deviates the optimal by the user-defined threshold, the portfolio will be rebalanced automatically before it moves to the next time step. |

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| Back test return performance over the investment horizon.  We can compare the performance against policy risk/return requirements to see whether this mix meets compliance requirements before the actual implementation. |

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| Historically simulated portfolio value distribution at the end of the investment horizon. |

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| Visually inspect some of the simulated scenarios. |

# Advantages and Disadvantages

Advantages:

* The codes are efficient. The total run time is approximately four minutes on the virtual machines. It includes both the optimization and thousands of historical simulations.
* The codes are well-structured and applies object-oriented-programming. The codes can be extended or modified easily in the future.
* Sharpe ratio optimization is based on a solid theoretical foundation and it connects to the Nobel-prize-winning modern portfolio theory.
* Historical simulations are based on actual events and objective. Basic historical simulations preserve cross correlations and serial correlations.
* This framework is straightforward and easy to be understood by someone who does not have a background in finance.
* The back testing performance can be compared against risk policies.

Disadvantages:

* Historical simulation fails to capture events that never happened in the history. History may not be the best representation of the future.
* Results depends on the chosen look back timeframe which can be subjective.
* Sharpe ratio optimization is based on return standard deviation which may not be the best risk measurement. Upside deviation is not a risk for some investors.
* Taxes and trading costs are not captured. Too frequent rebalancing can significantly impact the portfolio value.

# Conclusion

This documentation introduced an approach to allocate investment assets by optimizing the Sharpe ratio and back-testing the resulting mix using historical simulations. The model has been coded in Python language and hosted on Google Colab which is a cloud-based IDE. We walked through the codes in this documentation. For future research, we may consider other risk/reward measures and add additional asset classes for the analysis.

# Reference

https://en.wikipedia.org/wiki/Efficient\_frontier